Analysis and Design of Four-Port and Five-Port Microstrip Disc Circuits

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Abstract—A generalized method for evaluating the S-parameters of a multiport circular microstrip disc circuit is presented. The method is based on the planar circuit approach in which the two-dimensional Green's function of a circular segment is used. Three different applications of multiport circular disc structures are illustrated by designing, i) a broadband (44 percent bandwidth for $|S_{11}| < 0.2$) five-port network suitable for six-port network analyzers, ii) a broad-band three-way power divider with -3 dB, -6 dB, and -6 dB outputs, and iii) a four-port microstrip "cross-over." Experimental results verify the design methodology in all of these three cases.

I. Introduction

Multiport circular microstrip disc structures have found applications in filters [1], junction circulators [2], hybrids [3], and more recently as symmetrical five-port junctions for six-port network analyzers [4],[5]. However, available analytical and experimental results are frequently restricted to two-port networks only [1],[6].

The present paper discusses a method for analyzing a multiport circular disc structure. This method uses the planar circuit approach in which the Green's function for a circular segment [7] is employed to evaluate the impedance matrix of the network. The more familiar scattering parameters are then obtained by using standard Z-matrix to S-matrix transformation [7]. This approach has been used for designing a broad-band symmetrical five-port network suitable for six-port circuit with a bandwidth of about 44 percent (defined for $|S_{11}| < 0.2$). Also, two novel four-port circuits have been obtained by using the design methodology presented in this paper. A three-way power divider circuit with output powers of -3 dB, -6 dB, and -6 dB has been designed. Also, a symmetrical four-port microstrip "cross-over" circuit (with orthogonally located ports isolated from each other) has been designed. Details of these designs and experimental results are discussed in the following sections.

II. Method of Analysis

The two-dimensional planar circuit approach is used by replacing the physical disc diameter with an effective diameter [8] surrounded by a perfect magnetic wall.

The impedance matrix of a multiport circular disc is obtained from the available Green's function [7] for a circular segment with a magnetic wall. The elements of the Z-matrix are obtained as follows [7]:

$$Z_{ij} = \frac{1}{W_i W_j} \int_{W_i} \int_{W_j} G(s|s_0)\, ds_0\, ds \tag{1}$$

where W_i and W_j represent the widths of ports i and j, respectively, and the Green's function G for ports on the circumference is given by

$$G(a, \phi_i | a, \phi_j) = \frac{j\omega\mu d}{\pi} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \cos\{n(\phi_i - \phi_j)\}}{\left(a^2 - \dfrac{n^2}{k_{nm}^2}\right)(k_{nm}^2 - k^2)} \tag{2}$$

where ϕ_i and ϕ_j specify the location of the two ports i and j as shown in Fig. 1(a). The substrate parameters are: height d, permeability μ, and dielectric constant ϵ_r. The parameter σ_n is 1 when $n = 0$ and is equal to 2 otherwise. The effective radius of the disc is a and the wavenumber k is $\omega\sqrt{\mu_0 \epsilon_0 \epsilon_{rd} \mu_r}$ effective, where ϵ_{rd} is the dynamic dielectric constant [8] given by

$$\epsilon_{rd} = \frac{C_{\text{dyn}}(\epsilon = \epsilon_0 \epsilon_r)}{C_{\text{dyn}}(\epsilon = \epsilon_0)} \tag{3}$$

where C_{dyn} represents the dynamic capacitance of the disc. Z_{ij} is being calculated at frequency ω, and k_{nm} satisfy

$$\left.\frac{\partial}{\partial \rho} J_n(k_{nm}\rho)\right|_{\rho = a} = 0. \tag{4}$$

For any port i, the Z-matrix element Z_{ii} may be written as

$$Z_{ii} = \frac{2j\omega\mu d a^2}{\pi W_i^2} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n \{1 - \cos[2n \sin^{-1}(W_i/2a)]\}}{n^2\left(a^2 - \dfrac{n^2}{k_{nm}^2}\right)(k_{nm}^2 - k^2)}. \tag{5}$$

Off-diagonal terms of the impedance matrix are found to be

$$Z_{ij} = \frac{j\omega\mu d a^2}{\pi W_i W_j} \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\sigma_n}{n^2\left(a^2 - \dfrac{n^2}{k_{nm}^2}\right)(k_{nm}^2 - k^2)} \cdot \{\cos[n(\Delta_i - \Delta_j)] - \cos[n(\Delta_i + \Delta_j]\} \cos(n\phi_{ij}) \tag{6}$$

Manuscript received April 30, 1985; revised July 15, 1985.

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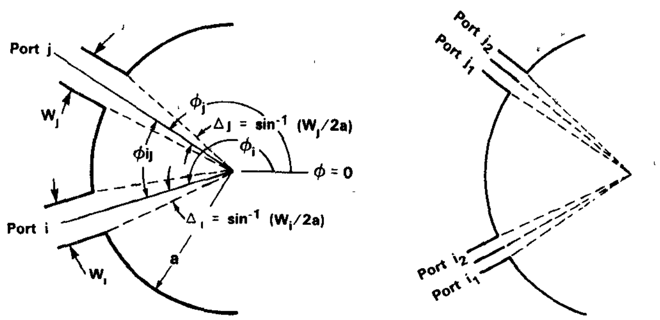


(a) (b)

Fig. 1. (a) Parameters for ports at the circumference of a circular disc. (b) Division of a wide port into subports.

where

$$\Delta_{i,j} = \sin^{-1}(W_{i,j}/2a).$$

Relations (5) and (6) are valid only when the widths W_i and W_j are small compared to wavelength $(2\pi c/\omega\sqrt{\epsilon_r})$ and, therefore, any field variation across W_i and W_j is negligible. At higher frequencies, or for wide ports, more accurate values of Z_{ii} and Z_{ij} are obtained by dividing each of the ports i and j into two (or more) subports as shown in Fig. 1(b). The widths of subports $i_1, i_2, \cdots$, etc., are now small for field variation along the port widths to be negligible. When the subports are being considered, the first step in the Z-matrix evaluation is to find the Z-matrix of the network with respect to various subports. For example, for a four-port network, this will be an 8×8 matrix if there are two subports for each of the four ports. The Z-matrix of the four-port is then derived as

$$[Z_{4p}] = [Y_{4p}]^{-1}. \tag{7}$$

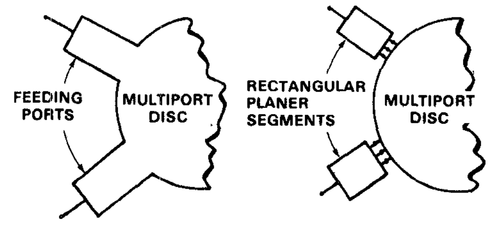
Elements of the admittance matrix Y_{4p} are given by [7]

$$Y_{ij} = \sum_{x = i_1, i_2, \cdots} \sum_{y = j_1, j_2, \cdots} Y_{xy} \tag{8}$$

where Y_{xy} are the elements of the admittance matrix of the networks considering each subport as an external port, and are found by evaluating the inverse of the corresponding Z-matrix.

In the procedure outlined above, the parasitic reactances at the junctions between the microstrip lines and the disc have not been considered. Effect of these reactances may be taken into account by including the feeding lines in the analysis of the planar circuit. For this purpose, small sections ($\approx \lambda/4$) of microstrip lines, adjacent to the junctions, are treated as planar rectangular segments as shown in Fig. 2(a). These rectangular segments are considered to be connected to the disc at a discrete number of interconnecting ports as shown in Fig. 2(b). The Z-matrix for these rectangular segments are then obtained by using the corresponding Green's functions [7]. Consequently, these matrices are combined with the Z-matrix of the disc, by using the segmentation method [7], and hence, an overall Z-matrix for the multiport network is obtained.

In the results reported in this paper, the frequency variation of Z_0 and the effective width of microstrip lines



(a) (b)

Fig. 2. (a) Configuration of the microstrip disc when the effects of junction reactances are included. (b) Modeling of the feeding lines as rectangular segments connected to the disc at discrete points.

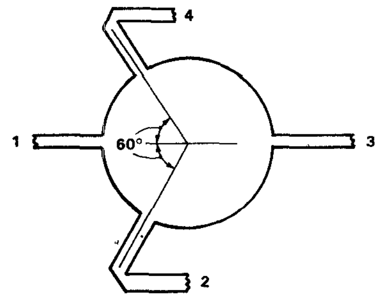


Fig. 3. Configuration of a three-way power divider.

has been taken into account by using the following relationships [9]:

$$W_{\text{eff}}(f) = \frac{120\pi d}{Z_0(f)\sqrt{\epsilon_{re}(f)}} \tag{9}$$

$$Z_0(f) = Z_0 - \frac{Z_{0T} - Z_0}{1 + G\left(\dfrac{f}{f_p}\right)^2} \tag{10}$$

$$\epsilon_{re}(f) = \epsilon_r - \frac{\epsilon_r - \epsilon_{re}}{1 + G\left(\dfrac{f}{f_p}\right)^2} \tag{11}$$

$$G = \left\{\frac{Z_0 - 5}{60}\right\}^{1/2} + 0.004\, Z_0 \tag{12}$$

where f_p (GHz) $= 15.66\, Z_0/h$ with Z_0 in ohms and h in mils, Z_{0T} is twice the characteristic impedance of a stripline of width W and ground-plane spacing $2h$, while Z_0 and ϵ_{re} are quasi-static values of the microstrip line characteristic impedance and the effective relative dielectric constant, respectively. More familiar S-parameters characterization is obtained by using standard Z-matrix to S-matrix transformation [7].

III. Design Examples and Experimental Results

The method of analysis discussed above has been used in an iterative computer-aided design process to explore various types of four-port and five-port circular microstrip disc structures. Three interesting design examples are discussed in this section.

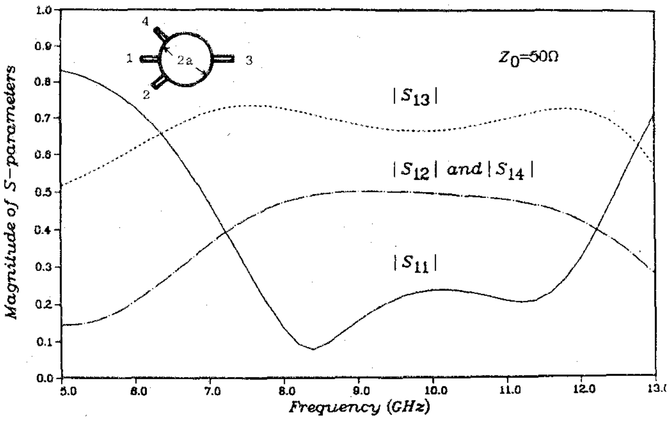


Fig. 4. Frequency characteristics of a circular disc power divider with $a = 7.60$ mm, $\epsilon_r = 2.20$, and $d = 1/32$ in. The characteristic impedance of the output ports is 50 Ω.

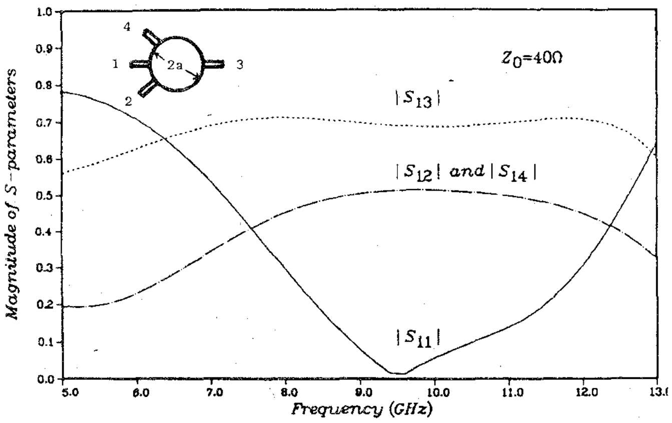


Fig. 5. Frequency characteristics of a circular disc power divider with $a = 7.60$ mm, $\epsilon_r = 2.20$, and $d = 1/32$ in. The characteristic impedance of the outgoing ports in 40 Ω.

A. Three-Way Power Divider

The analytical method discussed above has been used for designing a three-way power divider circuit. The circuit configuration is shown in Fig. 3. The coupling properties of this network are controlled by properly choosing the relative position of the outgoing ports around the disc circumference. By positioning ports 2, 3, and 4 at $\theta = -60°$, 180°, and 60°, respectively, it is possible to couple half the input power to port 3 and one fourth each to ports 2 and 4. Fig. 4 shows the calculated frequency response of this circuit which is designed on a 1/32-in thick substrate ($\epsilon_r = 2.20$). A bandwidth of 41 percent around 9.5 GHz is obtained. It should be noted here, that this frequency range is around the (1,1) mode and the (2,1) mode resonances which occur at 8.4 GHz and 11.2 GHz, respectively. Results of a computer-aided analysis reveal that the input reflection coefficient $|S_{11}|$ can be reduced by changing the characteristic impedance of the lines at various port to 40 Ω as shown in Fig. 5. The best $|S_{11}|$ value now is 0.0134 compared to the previous value of 0.078 for $Z_0 = 50$ Ω. Of course, this later arrangement would need external impedance transformation if the usual 50-Ω impedance level is desired.

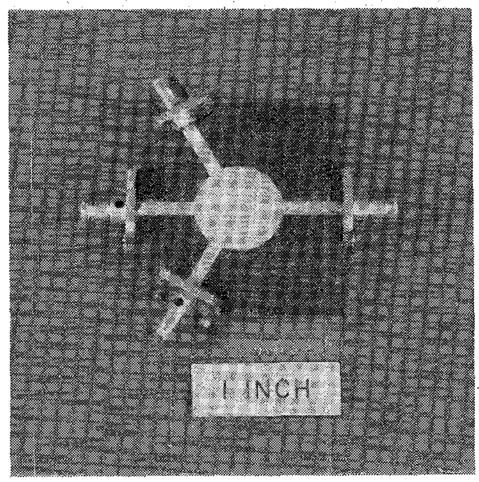


Fig. 6. Photograph of the experimental circular disc power divider fabricated on a Duroid substrate.

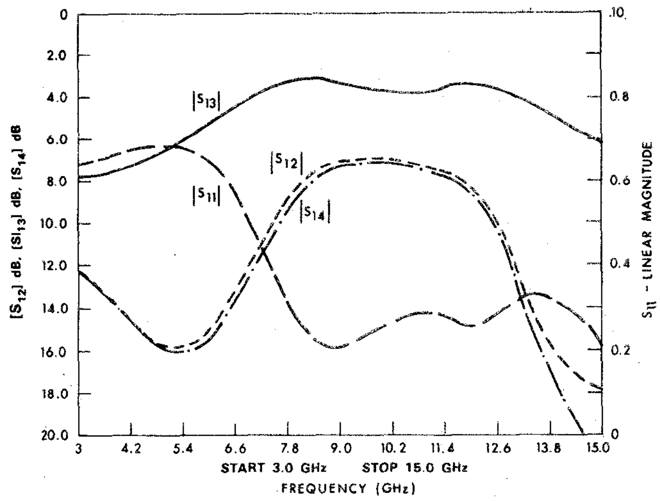


Fig. 7. Measured scattering parameters of the experimental three-way power divider model as a function of frequency ($a = 7.60$ mm, $\epsilon_r = 2.2$, $d = 1/32$ in).

In order to confirm the theoretical findings, a prototype of the power divider of Fig. 3 has been fabricated as shown in Fig. 6. This device is tested and the experimental results shown in Fig. 7 are in agreement with the theoretical results.

Circuit configurations leading to other power division ratios are also possible. Some preliminary results are shown in Table I, where θ represents the angle between port 1 and ports 2 and 4.

B. Microstrip "Cross-Over"

Another interesting circuit configuration is obtained by repositioning ports 2 and 4 at $-90°$ and $+90°$, respectively, as shown in the Fig. 9 insert. This symmetrical and reciprocal four-port network could serve as a "cross-over" junction for which $|S_{12}| = |S_{14}| = 0$ and $|S_{13}| = 1$. The theoretical behavior of this circuit is shown in Fig. 8 in which we observe the "cross-over" characteristics of the (1,1) mode and the (3,1) mode resonances. Zeros and maximas of the field distribution of these modes are also shown in

TABLE I

θ	P_1	P_2	P_4
45°	3/4	1/8	1/8
50°	2/3	1/6	1/6
60°	1/2	1/4	1/4

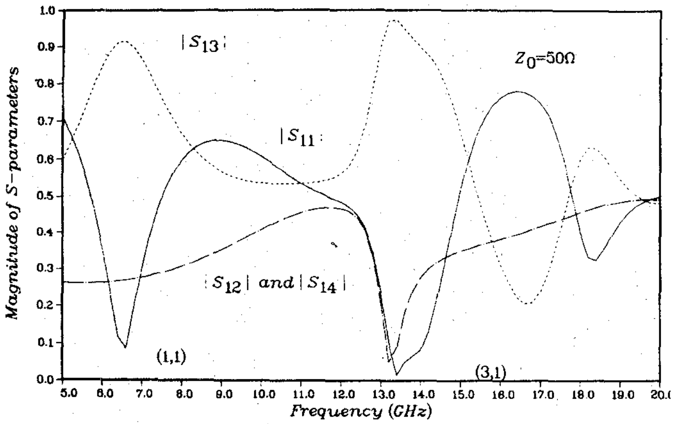


Fig. 8. Calculated scattering parameters of a circular disc "cross-over" circuit as a function of frequency. Characteristic impedance of the outgoing ports is 50 Ω ($a = 7.60$ mm, $\epsilon_r = 2.2$, $d = 1/32$ in).

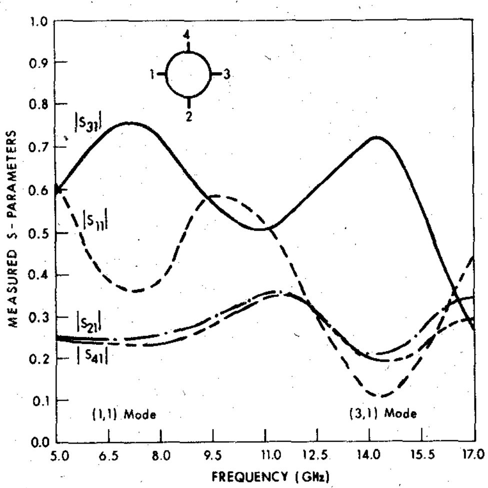


Fig. 9. Measured scattering parameters versus frequency for a microstrip "cross-over" junction.

Fig. 8. Ports located at zeros become isolated ports. Experimental results for a circuit fabricated on a 1/32-in-thick Duroid substrate with $\epsilon_r = 2.2$ are shown in Fig. 9. The experimental results are found to be in good, qualitative agreement with the calculated results. The measured "cross-over" characteristic of the (1,1) mode is not in good quantitative agreement with theory and is currently being investigated. Such a "cross-over" junction would be very useful in simplifying layout problems in complicated microwave integrated circuits.

C. Symmetrical Five-Port Junction

The same analytical method has been used for designing five-port networks in several frequency bands. Some typical calculated results are summarized in Figs. 10 and 11. Fig. 10 shows the frequency response of a five-port circuit designed on a 1/16 in-thick substrate ($\epsilon_r = 2.20$). In this case, the usable frequency range (defined for $|S_{11} < 0.2$ or VSWR < 1.5) extends from 9.4 GHz to 14.8 GHz, which yields a bandwidth of 44.63 percent around 12.1 GHz. By frequency scaling [7], this design can be modified to cover the complete X-band range. It may be noted here that this frequency range is around the (2,1) mode and the (0,1) mode of the circular disc resonator. Experimental results shown in Fig. 12 verify this design.

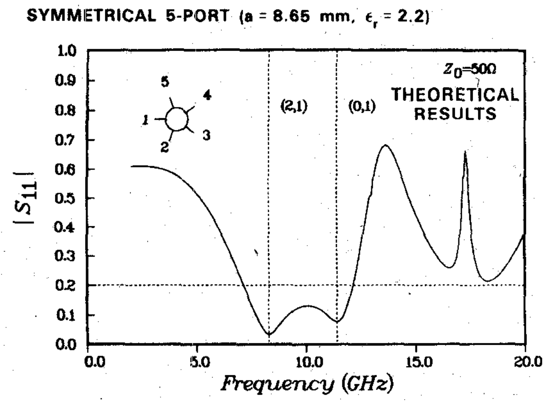


Fig. 10. $|S_{11}|$ for five-port circuit on 1/16-in-thick substrate.

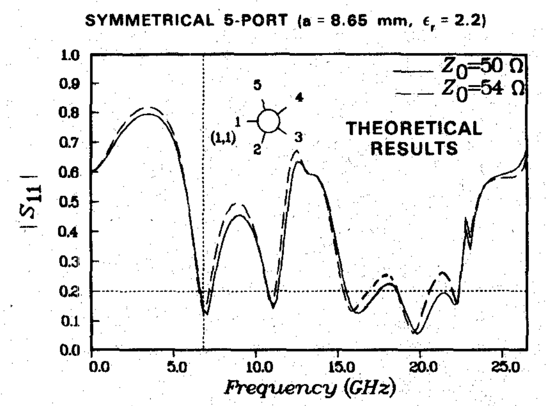


Fig. 11. $|S_{11}|$ for five-port circuit on 1/32-in-thick substrate.

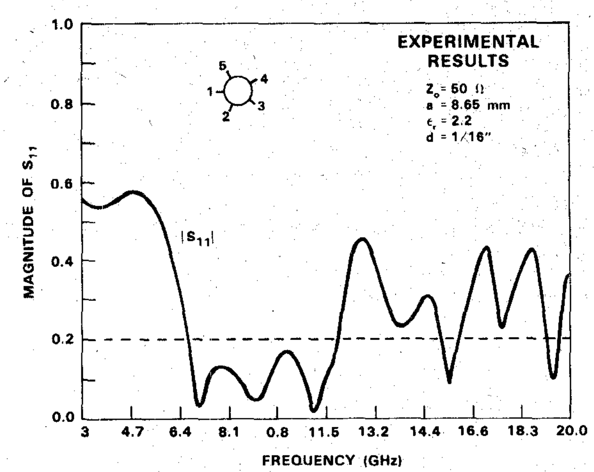


Fig. 12. Measured $|S_{11}|$ versus frequency for a five-port disc circuit with $a = 8.65$ mm, $d = 1/16$ in, $\epsilon_r = 2.2$.

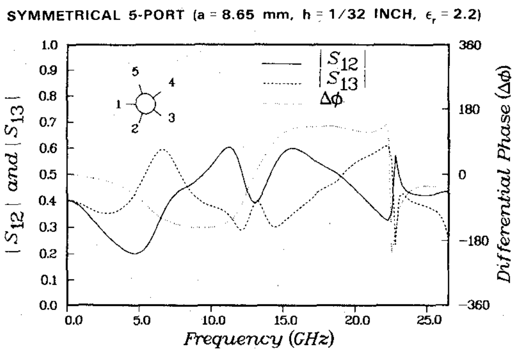


Fig. 13. Transmission coefficients $|S_{12}|$ and $|S_{13}|$, and port-to-port differential phase $\Delta\phi_{23}$ versus frequency for a symmetrical five-port microstrip circular disc.

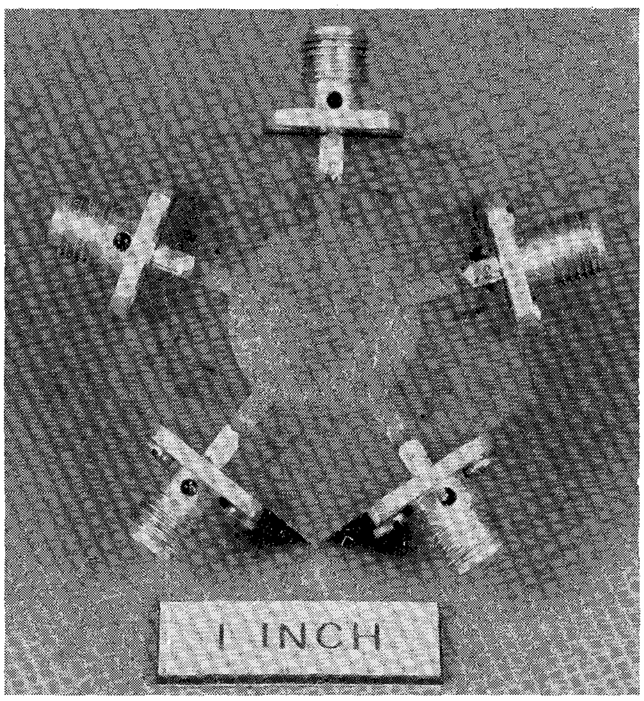


Fig. 14. Photograph of the experimental, five-port microstrip circular disc power divider fabricated on a 1/32-in-thick Duroid substrate.

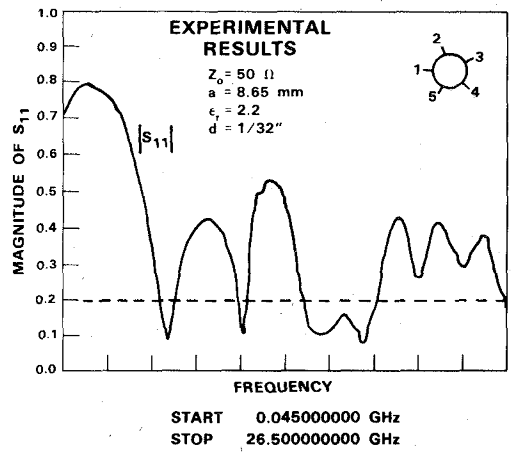


Fig. 15. Experimental behavior of the reflection coefficient $|S_{11}|$ versus the frequency for a five-port circular disc.

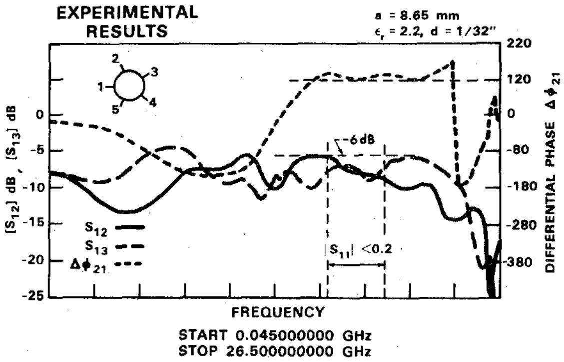


Fig. 16. Measured behavior of the transmission coefficients $|S_{12}|$ and $|S_{13}|$ and the differential phase between ports 2 and 3 versus frequency.

Five-port networks may also be designed for operation in a frequency band near the (5,1) mode of the circular disc resonator. Because of the larger physical size and, therefore, better tolerance characteristics, this design is preferable at millimeter-wave frequencies. The calculated input reflection coefficient $|S_{11}|$ for this design (on a 1/32-in-thick substrate) is shown in Fig. 11 for 50-Ω and 54-Ω impedances of the outgoing transmission lines. The 50-Ω circuit has a bandwidth of 9 GHz (44 percent), whereas, the 54-Ω has a bandwidth of 9.2 GHz.

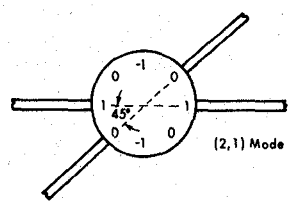
The calculated frequency dependence of S_{12}, S_{13}, and the differential phase shift between ports 2 and 3 are shown in Fig. 13. This type of five-port structure has been fabricated on a 1/32-in-thick dielectric substrate with $\epsilon_r =$ 2.20. A photograph of this prototype is shown in Fig. 14 and an outline of the circuit is included in Fig. 15.

Extensive measurements have been carried out using an Automatic Network Analyzer and the results are plotted in Figs. 15 and 16. Good agreement between the theoretical and experimental results confirms the validity of the design approach presented in this paper. The experiments reported above have been carried out with the impedance of the output ports equal to 50 Ω. Results of the computer-aided analysis has indicated that it is possible to increase the bandwidth of this network by modifying the impedance level of the outgoing ports.

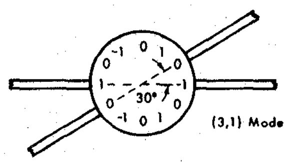
IV. Discussion

A method for analyzing multiport circular disc microstrip structures has been presented. This method has been used successfully for designing three different types of circuits. Even though the effects of junction discontinuity reactances were not incorporated in the design, the experimental results are seen to be in good agreement with the theoretical calculations. This verifies the validity of the approach discussed in this paper.

The three-way power divider reported here is a novel design concept and should be a useful candidate for several power distributing or power-combining subsystems. The bandwidth of 41 percent reported for the -3 dB, -6 dB, -6 dB circuit (discussed in detail in Section III) is obtained as a result of the interaction between the (1,1) and (2,1) modes of resonances in circular disc structures. This large bandwidth is attractive for several applications. It is quite



(a)



(b)

Fig. 17. (a) Voltage distribution for the (2,1) resonant mode of a four-port circular disc circuit. (b) Voltage distribution for the (3,1) resonant mode of a four-port circular disc circuit.

likely that the basic design idea of the three-way power division, reported in this paper, can be extended for four-way (or more than four-way) power division circuits.

The microstrip cross-over circuit discussed in Section III-B operates in (1,1) and (3,1) modes. Similar cross-over circuits may also be designed to operate around the resonance frequencies of other modes of circular disc. For example, the (2,1) mode resonance can be used when the microstrip lines are crossing each other at an angle of 45°, as shown in Fig. 17(a). Also, the (3,1) mode resonance can also be used when the angle between the crossing microstrip lines is only 30° as shown in Fig. 17(b). System applications of these cross-overs would perhaps be limited because of their narrow bandwidths.

The bandwidth performance of the symmetrical five-port circuit discussed in Section III-C is better than that for similar circuits reported earlier [4],[5]. It may be noted that no tuning or broad-banding mechanisms have been used in the design reported here.

Although an iterative computer-aided analysis process has been used for the design examples reported in this paper, these designs are not decidedly optimum. A computer-aided optimization procedure can be built in the design methodology presented here by evaluating the sensitivities of Z-parameters with respect to various designable parameters (disc diameter, location, and widths of the ports, etc.) and using a gradient optimization technique. Thus, the design procedure is suitable for CAD implementation.

The planar circuit approach described in this paper can also be extended to the analysis and design of corresponding reduced-height waveguide circuits consisting of a thin, cylindrical cavity (in place of the microstrip disc) and reduced-height waveguides as the feeding lines (in place of microstrip lines). Analysis of such reduced-height waveguide circuits may be carried out by replacing the Green's function used in this paper by the corresponding Green's function for a circular segment with an electric wall around the circumference.

Acknowledgment

Many thanks are due to P. Mosher for typing the manuscript. The authors are also indebted to L. Kushner for his assistance in the measurements.

Mr. Abouzahra is also indebteded to S. Al-Marzouk for his support and encouragement.

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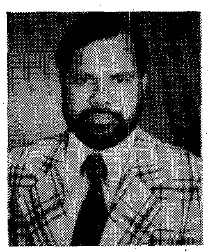

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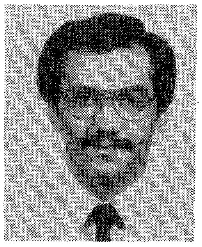

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